Fuzzy Syllogistic Reasoning with Generalized Quantifiers

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What is a syllogism?

Classical syllogism is a deductive inference schema made up of three quantified statements (categorical statements) and three terms (Major Term, Minor Term and Middle Term). Two of the statements are the premises and the other one, the conclusion.

An example of syllogism

PR1	All human beings are mortal
PR2	All Greeks are human beings
C	All Greeks are mortal

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Involved fields

- Argumentation theory: Interdisciplinary study of how conclusions can be reached through logical reasoning; that is, claims based, soundly or not, on premises.
  - It includes the arts and sciences of civil debate, dialogue, conversation, and persuasion.
  - It studies rules of inference, logic, and procedural rules in both artificial and natural languages.
- Theory of Generalized Quantifiers: The current standard theory in linguistics about the quantification phenomenon in natural language. It defines a generalized quantifier as an expression that denotes a property of a property, also called a higher-order property.

Every boy sleeps

$$\{x|x \text{ is a boy}\} \in \{x|x \text{ sleeps}\}$$

Classical Syllogistics

- Developed by Aristotle.
- Categorical statements Q A are B:
  - Q stands for one of the four classical crisp quantifiers → All, No, Some, Some. . . not.
  - A stands for the subject-term or restriction → crisp set.
  - B stands for the predicate-term or scope → crisp set.
  - are stands for the corresponding form of the copulative verb → to be.

Limitations

1. It cannot manage vague quantifiers like many, most, few,. . .
2. It cannot deal with arguments with more than two premises.
3. It cannot manage vague terms like tall,. . . or synonyms like car-automobile,. . .

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Proposals to overcome these limitations

- Introducing new quantifiers:
  - Peterson, 2000: proportional quantifiers like most, few, almost all,... with crisp definitions.
  - Zadeh, 1985; Dubois, 1988; Novak, 2008: proportional quantifiers like most, few, almost all,... with fuzzy definitions.

$$\begin{array}{l} \text{Most students are tall} \\ \text{Most tall students are blond} \\ \hline \text{Most} \otimes \text{Most students are tall and blond} \end{array}$$

- Increasing the number of premises:
  - Sommers, 1982: it can manage arguments with n premises and n terms.

$$\begin{array}{l} \text{Some animal is a pet} \\ \text{All non-domestic animals are non-pets} \\ \text{No animal is non-mortal} \\ \hline \text{Some domestic animal is mortal} \end{array}$$

Proposals to overcome these limitations

Limitations

- Only proportional quantifiers are managed (most, few, many,...). Other quantifiers like exception quantifiers (all but three,...) or comparative quantifiers (double, half,...) are not considered.
- Proposals that manage vague quantifiers and involving n premises are not considered.
- Fuzzy approaches cannot adequately manage the classical syllogisms[1].

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Working hypothesis

Is it possible to develop a general approach to syllogism that can deal with arguments involving fuzzy quantifiers and $n > 2$ premises?

Objectives

$$\begin{array}{rl} \mathrm{PR1}: & \mathrm{Q_1\ L_{1,1}\ are\ L_{1,2}} \\ \mathrm{PR2}: & \mathrm{Q_2\ L_{2,1}\ are\ L_{2,2}} \\ \ldots & \\ \mathrm{PRN}: & \mathrm{Q_N\ L_{N,1}\ are\ L_{N,2}} \\ \hline \mathrm{C}: & \mathrm{Q_C\ L_{C,1}\ are\ L_{C,2}} \end{array}$$

1. $\mathrm{Q_i}$ is a quantifier of the Theory of Generalized Quantifiers.
2. $\mathrm{L_{i,j}}$ is also any boolean combination of the term sets.
3. The argument can have any number of premises an terms.

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Looking through Aristotle's eyes

What is the meaning of a categorical statement?

A relation of quantity between sets

What is a syllogism?

A reasoning schema based on the quantity relations among sets

How would be an "extended syllogistics"?

A systematic method for inferring the quantifier of the conclusion in terms of the quantifiers of the premises

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Our proposal

Transforming the syllogistic reasoning process into an optimization problem where the quantifiers can be:

- crisp ≡ precise quantity: five, 50 %. . .
- interval ≡ imprecise quantity but well-defined bounds: between five and seven, between 50 % and 70 %,. . .
- fuzzy ≡ fuzzy quantity with imprecise-defined bounds: around five, something more than 50 %,. . .

Our proposal

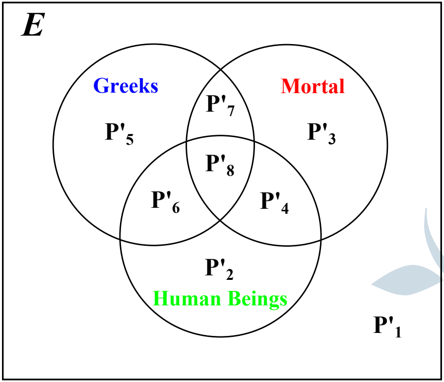
This process is divided into three steps:

- Division of the reference universe in disjoint sets.
- Definition of the quantifiers according to Theory of Generalized Quantifiers as sets of inequations.
- Application of a resolution method to the equivalent optimization problem, using methods of linear and fractional programming.

Division of the reference universe in disjoint sets

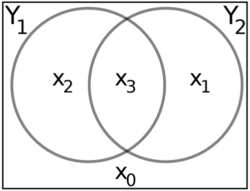
Example

Greeks$= \mathrm{P}'_5 \cup \mathrm{P}'_6 \cup \mathrm{P}'_7 \cup \mathrm{P}'_8$, Not human beings and mortal$= \mathrm{P}'7 \cup \mathrm{P}'_3, \dots$



Df. of the quantifiers through inequations

- $x_0 = \overline{Y_1} \cap \overline{Y_2}$
- $x_1 = \overline{Y_1} \cap Y_2$
- $x_2 = Y_1 \cap \overline{Y_2}$
- $x_3 = Y_1 \cap Y_2$



Logic Quantifiers		Inequations
some (Y_1,Y_2)	$0 : Y_1 \cap Y_2 = \emptyset$ $1 : Y_1 \cap Y_2 \neq \emptyset$	$x_3 > 0$
all (Y_1,Y_2)	$0 : Y_1 \not\subseteq Y_2$ $1 : Y_1 \subseteq Y_2$	$x_2 = 0$
no (Y_1,Y_2)	$0 : Y_1 \cap Y_2 \neq \emptyset$ $1 : Y_1 \cap Y_2 = \emptyset$	$x_3 = 0$
not all (Y_1,Y_2)	$0 : Y_1 \subseteq Y_2$ $1 : Y_1 \not\subseteq Y_2$	$x_2 > 0$

Df. of the quantifiers through inequations

- $x_0 = \overline{Y_1} \cap \overline{Y_2}$
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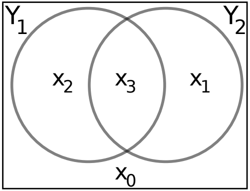


Absolute Binary Quantifiers		Inequations
$Q_{BA}(Y_1, Y_2)$	$0 : \lvert Y_1 \cap Y_2 \rvert \notin [a, b]$ $1 : \lvert Y_1 \cap Y_2 \rvert \in [a, b]$	$x_3 \geq a; x_3 \leq b$
Exception Binary Quantifiers		
$Q_{BE}(Y_1, Y_2)$	$0 : \lvert Y_1 \cap \overline{Y_2} \rvert \notin [a, b]$ $1 : \lvert Y_1 \cap \overline{Y_2} \rvert \in [a, b]$	$x_2 \geq a; x_2 \leq b$
Proportional binary quantifiers		
$Q_{PB}(Y_1, Y_2) =$	$0 : \frac{\lvert Y_1 \cap Y_2 \rvert}{\lvert Y_1 \rvert} \notin [a, b]$ $1 : \frac{\lvert Y_1 \cap Y_2 \rvert}{\lvert Y_1 \rvert} \in [a, b]$ $1 : \lvert Y_1 \rvert = 0$	$\frac{x_3}{x_2 + x_3} \geq a; \frac{x_3}{x_2 + x_3} \leq b$

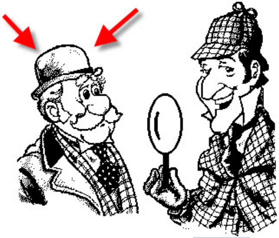
Resolution Method for the Equivalent Optimization Problem

- Formalize the premises as the corresponding set of inequations.
- Three additional constraints must be added;
  - there are not empty sets or sets with a negative cardinality, $x_k \geq 0, \forall k = 0, ..., 2^S - 1$
  - the $L_{n,j}$ are not empty, to avoid indefinition in the results, $x_1^{n,j} + \cdots + x_r^{n,j} > 0, \forall n = 1, ..., N$
  - the addition of the cardinalities equals to the size of the universe, $\sum_{k=0}^{K} x_k = |E|$
- Optimize the conclusion of the syllogism, that will be formalized accordingly to the corresponding quantifier, using tecniques of linear or fractional-programming [2].

Wine warehouse

Order

For tomorrow, all but around fifteen boxes of wine must be send to J. Moriarty. On the other hand, do not forget preparing around four boxes of the remaining ones for S. Holmes. Finally, tell me how many boxes remain in the warehouse.



Example of syllogism

PR1 : All but around fifteen boxes of wine are for J. Moriarty
PR2 : Around four boxes of those that are not for J. Moriarty are for S. Holmes

C : Q_C boxes are not for J. Moriarty neither for S. Holmes

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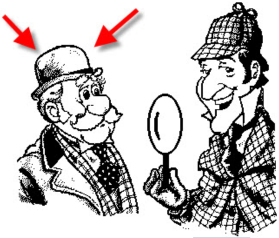

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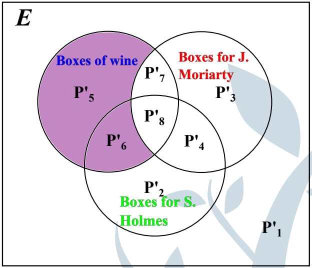
C : Q_C boxes are not for J. Moriarty neither for S. Holmes

Wine warehouse

- Sets: boxes of wine, boxes for J. Moriarty and boxes for S. Holmes
- Quantifiers: $Q_1 = [\tilde{15}]$, $Q_2 = [\tilde{4}]$ and $Q_C = [a, c, d, b]$.

Transformation into an equivalent set of inequations:

$$\begin{array}{rrr}
PR1_s: & |P'_5| + |P'_6| \geq 14; & |P'_5| + |P'_6| \leq 16 \\
PR1_k: & |P'_5| + |P'_6| \geq 15; & |P'_5| + |P'_6| \leq 15 \\
PR2_s: & |P'_6| \geq 3; & |P'_6| \leq 5 \\
PR2_k: & |P'_6| \geq 4; & |P'_6| \leq 4 \\
C_s: & |P'_5| \geq a\ ; & |P'_5| \leq b \\
C_k: & |P'_5| \geq c\ ; & |P'_5| \leq d
\end{array}$$



Result

$Q = [9, 11, 11, 13]$

Around eleven boxes remain in the warehouse.

Wine warehouse

- Sets: boxes of wine, boxes for J. Moriarty and boxes for S. Holmes
- Quantifiers: $Q_1 = [\tilde{15}]$, $Q_2 = [\tilde{4}]$ and $Q_C = [a, c, d, b]$.

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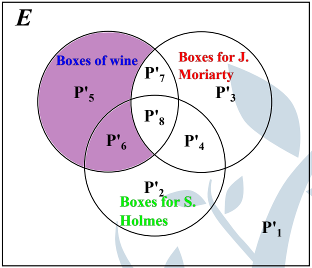
PR1_k : &\quad |P'_5| + |P'_6| \geq 15; &\quad |P'_5| + |P'_6| \leq 15 \\
PR2_s : &\quad |P'_6| \geq 3; &\quad |P'_6| \leq 5 \\
PR2_k : &\quad |P'_6| \geq 4; &\quad |P'_6| \leq 4 \\
C_s : &\quad |P'_5| \geq a\ ; &\quad |P'_5| \leq b \\
C_k : &\quad |P'_5| \geq c\ ; &\quad |P'_5| \leq d
\end{aligned}$$



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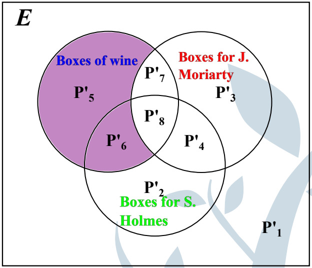
PR1_k: & |P'_5| + |P'_6| \geq 15; & |P'_5| + |P'_6| \leq 15 \\
PR2_s: & |P'_6| \geq 3; & |P'_6| \leq 5 \\
PR2_k: & |P'_6| \geq 4; & |P'_6| \leq 4 \\
C_s: & |P'_5| \geq a\ ; & |P'_5| \leq b \\
C_k: & |P'_5| \geq c\ ; & |P'_5| \leq d
\end{array}$$



Result

$Q = [9, 11, 11, 13]$

Around eleven boxes remain in the warehouse.

Riddle: Dogs, cats and parrots

Order

Dogs, cats and parrots. How many animals do I have in my home, if all but two are dogs, all but two are cats and all but two are parrots?

Example of syllogism

PR1 :	All animals but two are dogs
PR2 :	All animals but two are cats
PR3 :	All animals but two are parrots
C :	There are Q_C animals in my home

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Dogs, cats and parrots. How many animals do I have in my home, if all but two are dogs, all but two are cats and all but two are parrots?

Example of syllogism

$$\begin{array}{rl} \mathrm{PR1}: & \text{All animals but two are dogs} \\ \mathrm{PR2}: & \text{All animals but two are cats} \\ \mathrm{PR3}: & \text{All animals but two are parrots} \\ \hline \mathrm{C}: & \text{There are } \mathrm{Q_C} \text{ animals in my home} \end{array}$$

Riddle: Dogs, cats and parrots

- Sets: dogs, cats, parrots and animals
- Quantifiers: $Q_1 = [\text{all} - 2]$, $Q_2 = [\text{all} - 2]$, $Q_3 = [\text{all} - 2]$ and $Q_C = [a, c, d, b]$.

Transformation into an equivalent set of inequations:

$$
\begin{aligned}
PR1 &: x_1 + x_2 + x_3 + x_4 = 2; \\
PR2 &: x_1 + x_2 + x_5 + x_6 = 2; \\
PR3 &: x_1 + x_3 + x_5 + x_7 = 2; \\
C &: x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 = a;
\end{aligned}
$$

Result

$Q = [2, \inf]$

There between two and infinite animals in my home.

Riddle: Dogs, cats and parrots

- Sets: dogs, cats, parrots and animals
- Quantifiers: $Q_1 = [\text{all} - 2]$, $Q_2 = [\text{all} - 2]$, $Q_3 = [\text{all} - 2]$ and $Q_C = [a, c, d, b]$.

Transformation into an equivalent set of inequations:

$$\begin{aligned} PR1 :& x_1 + x_2 + x_3 + x_4 = 2; \\ PR2 :& x_1 + x_2 + x_5 + x_6 = 2; \\ PR3 :& x_1 + x_3 + x_5 + x_7 = 2; \\ C :& x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 = a; \end{aligned}$$

Result

$Q = [2, \inf]$
There between two and infinite animals in my home.

Riddle: Dogs, cats and parrots

Syllogism with implicit premises

PR1 :	All animals but two are dogs
PR2 :	All animals but two are cats
PR3 :	All animals but two are parrots
PR4 :	No dog, cat or parrot is not an animal
PR5 :	No animal is not a dog, a cat or a parrot
PR6 :	No dog is a cat or a parrot
PR7 :	No cat is a dog or a parrot
PR8 :	No parrot is a dog or a cat
C :	There are Q_C animals in my home

Result

$Q = [3, 3]$ There three animals in my home.

Riddle: Dogs, cats and parrots

Syllogism with implicit premises

PR1 :	All animals but two are dogs
PR2 :	All animals but two are cats
PR3 :	All animals but two are parrots
PR4 :	No dog, cat or parrot is not an animal
PR5 :	No animal is not a dog, a cat or a parrot
PR6 :	No dog is a cat or a parrot
PR7 :	No cat is a dog or a parrot
PR8 :	No parrot is a dog or a cat
C :	There are Q_C animals in my home

Result

$Q = [3, 3]$ There three animals in my home.

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Strengths and Weaknesses

Strenghts

- It can deal with syllogisms involving fuzzy quantifiers and without predefined number of premises and terms.
- It manage natural language and it is compatible with the Theory of Generalized Quantifiers.
- It can manage more quantifiers than the usual distinction absolute/proportion (exception, comparative,...).
- FARMALIB

Weaknesses

- It manage rather semi-fuzzy quantifiers than fuzzy ones.
- It does not have an associated semantics.
- It cannot manage simultaneously proportional and no-proportional quantifiers.
- The effects of considering approximate terms instead exact ones is not analyzed in this model.

Future work and collaborations

- Improving the pointed out weaknesses.
  - Adding an underlying semantics to the definition of the quantifiers.
- Extend a FARMALIB for its experimental use.
- Applications in contexts where syllogistic reasoning can be useful; i.e. reasoning layer in linguistic summarization of data.
- Its combination with other reasoning mechanims: Mamdani, TSK,...

M. Pereira-Fari na, Juan C. Vidal-Aguiar, P. Montoto, F. Díaz-Hermida, and A. Bugarín.
Razonamiento silogístico aproximado con cuantificadores generalizados.
In Actas del XVI Congreso Español de Tecnologías y Lógica Fuzzy (ESTYLF2012), pages 701–706, 2012.

S. Boyd and L. Vandenberghe.
Convex Optimization.
Cambridge University Press, The Edinburgh Building, Cambridge, UK, 2004.